DAY TWENTY NINE

Wave Optics

Learning & Revision for the Day

- Wavefront
- Huygens' Principle
- Interference of Light
- Young's Double Slit Experiment
- Coherent Sources
- Interference in Thin Films
- Diffraction
- Polarisation of Light
- Brewster's Law
- Polaroids

According to Huygens', light is a form of energy, which travels in the form of waves through a hypothetical medium 'ether'. The medium was supposed to be all pervading, transparent, extremely light, perfectly elastic and an ideal fluid.

Light waves transmit energy as well as momentum and travel in the free space with a constant speed of $3 \times 10^8 \text{ ms}^{-1}$. However, in a material medium, their speed varies from medium to medium depending on the refractive index of the medium.

Wavefront

A wavefront is the locus of all those points (either particles) which are vibrating in the same phase. The shape of the wavefront depends on the nature and dimension of the source of light.

- In an isotropic medium, for a point source of light, the wavefront is spherical in nature.
- For a line (slit) source of light, the wavefront is cylindrical in shape.
- For a parallel beam of light, the wavefront is a plane wavefront.

Huygens' Principle

Every point on a given wavefront, acts as secondary source of light and emits secondary wavelets which travel in all directions with the speed of light in the medium. A surface touching all these secondary wavelets tangentially in the forward direction, gives the new wavefront at that instant of time.

Laws of reflection and refraction can be determined by using Huygens' principle.

Interference of Light

Interference of light is the phenomenon of redistribution of light energy in space when two light waves of same frequency (or same wavelength) emitted by two coherent sources, travelling in a given direction, superimpose on each other. If a_1 and a_2 be the amplitudes of two light waves of same frequency and ϕ be the phase difference between them, then the amplitude of resultant wave is given by

$$A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

and in terms of intensity of light,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi.$$

Condition for Constructive Interference

If at some point in space, the phase difference between two waves, $\phi = 0°$ or $2n\pi$ or path difference between two waves, $\Delta = 0$ or $n\lambda$, where n is an integer, then $A_R = a_1 + a_2$ or $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}$ is maximum. Such an interference is called **constructive interference**.

Condition for Destructive Interference

If at some point in space, the phase difference between two waves, $\phi = (2n-1)\pi$ or path difference, $\Delta\delta = (2n-1)\dfrac{\lambda}{2}$, then at such points $A_R = (a_1 - a_2)$ and $I_R = I_1 + I_2 - 2\sqrt{I_1 I_2}$ is minimum leading to a **destructive interference**.

Amplitude Ratio

$$\frac{I_{\max}}{I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2$$

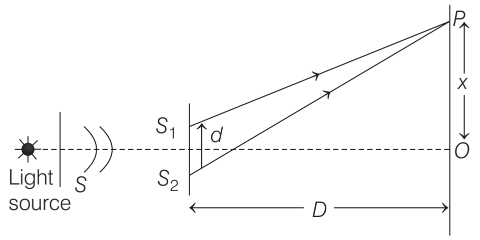
$$= \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{r+1}{r-1}\right]^2$$

where, $r = \dfrac{a_1}{a_2}$ = amplitude ratio.

NOTE
- For identical sources, $I_1 = I_2 = I_0$
- For constructive interference, $I_{\max} = 4I_0$ and $I = 4I_0 \cos^2 \dfrac{\phi}{2}$
- For destructive interference, $I_{\min} = 0$

Young's Double Slit Experiment

The arrangement of young's double slit experiment is shown in figure. Here monochromatic light of one wavelength is used.



Bright and dark fringes are formed on the screen with central point O be having as the central bright fringe, because for O, the path difference $\Delta = 0$.

For light waves reaching a point P, situated at a distance x from central point Δ, the path difference,

$$\Delta = S_2 P - S_1 P = \frac{xd}{D}$$

Case I If $\dfrac{xd}{D} = n\lambda$, then we get nth bright fringe. Hence, position of bright fringes on the screen are given by the relation, $x = \dfrac{nD\lambda}{d}$.

Case II If $\dfrac{xd}{D} = (2n-1)\dfrac{\lambda}{2}$, then we get nth dark fringe.

Hence, for nth dark fringe, $x = \dfrac{(2n-1)D\lambda}{2d}$

where, $n = 1, 2, 3, \ldots\ldots$.

Fringe Width

The separation between any two consecutive bright or dark fringes is called fringe width β.

Thus, $$\beta = \frac{D\lambda}{d}$$

and for a given arrangement, it is constant, i.e. all fringes are uniformly spaced.

Moreover, fringe width β is

(i) $\beta \propto D$, (ii) $\beta \propto \lambda$ and (iii) $\beta \propto \dfrac{1}{d}$.

Angular fringe width of interference pattern,

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$$

If in a given field of view n_1, fringes of light of wavelength λ_1 are visible and n_2 fringes of wavelength λ_2 are visible, then $n_1\lambda_1 = n_2\lambda_2$

NOTE
- If whole apparatus of Young's double slit experiment is immersed in a transparent medium of refractive index n_m, then fringe width in the medium, $\beta_m = \dfrac{D\lambda}{n_m d}$.

Coherent Sources

Two light sources are said to be coherent, if they emit light of exactly same frequency (or wavelength), such that the originating phase difference between the waves emitted by them is either zero or remains constant. For sustained interference pattern, the interfering light sources must be coherent one.

There are two possible techniques for obtaining coherent light sources.

- In division of wavefront technique, we divide the wavefront emitted by a narrow source in two parts by reflection, refraction or diffraction.
- In division of amplitude technique, a single extended light beam of large amplitude is splitted into two or more waves by making use of partial reflection or refraction.

NOTE
- Two independent sources of light can never be coherent. Two light sources can be coherent only, if these have been derived from a single parental light source.

Interference in Thin Films

In white light thin films, whose thickness is comparable to wavelength of light, show various colours due to interference of light waves reflected from the two surfaces of thin film.

For interference in reflected light condition of constructive interference (maximum intensity),

$$\Delta = 2n_m\, t \cos r = (2n-1)\,\frac{\lambda}{2}$$

Condition of destructive interference
(minimum intensity),

$$\Delta = 2n_m\, t \cos r = (2n)\,\frac{\lambda}{2}$$

For interference in refracted light condition of constructive interference (maximum intensity),

$$\Delta = 2n_m t \cos r = (2n)\,\frac{\lambda}{2}$$

Condition of destructive interference (minimum intensity),

$$\Delta = 2n_m t \cos r = (2n-1)\,\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots$$

Shift in Interference Pattern

If a transparent thin sheet of thickness t and refractive index n_m is placed in the path of one of the superimposing waves (say in front of slit S_2 of Young's double slit experiment), then it causes an additional path difference due to which interference pattern shifts.

- Additional path difference due to sheet $= (n_m - 1)\, t$
- Fringe shift $= \dfrac{D}{d}(n_m - 1)t = \dfrac{\beta}{\lambda}(n_m - 1)t$
- If due to presence of thin film, the fringe pattern shifts by n fringes, then

$$n = \frac{(n_m - 1)\, t}{\lambda}$$

or

$$t = \frac{n\lambda}{(n_m - 1)}$$

Shift is independent of the order of fringe and wavelength.

NOTE
- **Fresnel's biprism** is a device to produce coherent sources by division of wavefront,

$$d = 2a(n-1)\alpha$$

The distance between the coherent sources and screen,

$$D = a + b$$

The fringe width is given by $\beta = \dfrac{D\lambda}{d} = \dfrac{\lambda(a+b)}{2a(n-1)\alpha}$

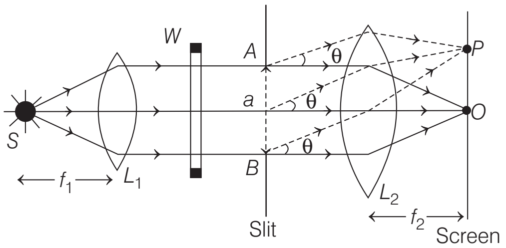
Diffraction

Diffraction of light is the phenomenon of bending of light around the edges of an aperture or obstacle and entry of light even in the region of geometrical shadow, when size of aperture or obstacle is comparable to wavelength of light used.

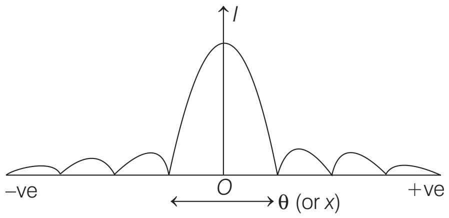
Diffraction is characteristic of all types of waves. Greater the wavelength, more pronounced is the diffraction effect. It is due to this reason that diffraction effect is very commonly observed in sound.

Diffraction due to Single Slit and Width of Central Maximum

Fraunhofer's arrangement for studying diffraction at a single narrow slit (width $= a$) is shown in adjoining figure. Lenses L_1 and L_2 are used to render incident light beam parallel and to focus parallel light beam.



As a result of diffraction, we obtain a broad, bright maxima at symmetrical centre point O and on either side of it, we get secondary diffraction maxima of successively falling intensity and poor contrast, as shown in figure.



- Condition of diffraction minima is given by

$$a \sin\theta = n\lambda$$

where, $n = 1, 2, 3, 4, \ldots$

But the condition of secondary diffraction maxima is

$$a \sin\theta = (2n+1)\,\frac{\lambda}{2}$$

where, $n = 1, 2, 3, 4, \ldots$

- Angular position of nth secondary minima is given by

$$\sin\theta = \theta = n\,\frac{\lambda}{a}$$

and linear distance, $x_n = D\theta = \dfrac{nD\lambda}{a} = \dfrac{nf_2\lambda}{a}$

where, f_2 is focal length of lens L_2 and $D = f_2$.

- Similarly, for nth maxima, we have

$$\sin\theta = \theta = \frac{(2n+1)\,\lambda}{2a} \text{ and } x_n = \frac{(2n+1)\,D\lambda}{2a} = \frac{(2n+1)\,f_2\lambda}{2a}$$

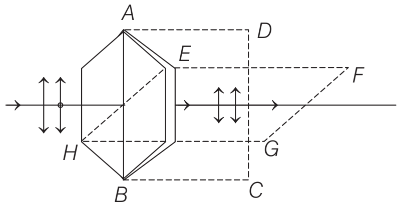
The angular width of central maxima is $2\theta = \dfrac{2\lambda}{a}$

or linear width of central maxima $= \dfrac{2D\lambda}{a} = \dfrac{2f_2\lambda}{2}$

NOTE
- The angular width of central maxima is double as compared to angular width of secondary diffraction maxima.

Polarisation of Light

- Light is an electromagnetic wave in which electric and magnetic field vectors very sinusoidally, perpendicular to each other as well as perpendicular to the direction of propagation of wave of light.
- The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to the direction of wave motion is called **polarisation of light**. The tourmaline crystal acts as a **polariser**.



Polarisation of Light

Thus, electromagnetic waves are said to be polarised when their electric field vector are all in a single plane, called the plane of oscillation/vibration. Light waves from common sources are upolarised or randomly polarised.

Plane Polarised Light

The plane *ABCD* in which the vibrations of polarised light are confined is called the **plane of vibration**. It is defined as The light, in which vibrations of the light (vibrations of electric vector) when restricted to a particular plane the light itself is called plane polarised light. The vibrations in a plane polarised light are perpendicular to the plane of polarisation.

NOTE
- Only transverse waves can be polarised. Thus, it proved that light waves are transverse waves.

Brewster's Law

According to this law, when unpolarised light is incident at an angle called polarising angle, i_p on an interface separating air from a medium of refractive index μ, then the reflected light is fully polarised (perpendicular to the plane of incidence), provided

$$\mu = \tan i_p$$

This relation represents Brewster's law. Note that the parallel components of incident light do not disappear, but refract into the medium, with the perpendicular components.

Law of Malus

When a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of cosine of angle (θ) between plane of transmission of analyser and polariser.

i.e. $$I \propto \cos^2 \theta$$

If intensity of plane polarised light incidenting on analyser is I_0, then intensity of emerging light from analyser is $I_0 \cos^2 \theta$.

NOTE
- We can prove that when unpolarised light of intensity I_0 gets polarised on passing through a polaroid, its intensity becomes half, i.e. $I = \frac{1}{2} I_0$.

Polaroids

Polaroids are thin and large sheet of crystalline polarising material (made artifically) which are capable of producing plane polarised beams of large cross-section.

The important uses are

- These reduce excess glare and hence sun glasses are fitted with polaroid sheets.
- These are also used to reduce headlight glare of cars.
- They are used to improve colour contrast in old oil paintings.
- In wind shields of automobiles.
- In window panes.
- In three dimensional motion pictures.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are
(a) $5I$ and I (b) $5I$ and $3I$ (c) $9I$ and I (d) $9I$ and $3I$

2 If the equations of two light waves are $y_1 = 8 \sin \omega t$ and $y_2 = 6 \sin (\omega t + \phi)$. Then, ratio of maximum and minimum intensity will be
(a) 11: 49 (b) 49 : 1
(c) 7 : 1 (d) 1: 7

3 The ratio of intensity at the centre of a bright fringe to the intensity at a point distance one-fourth of the distance between two successive bright fringes will be
(a) 4 (b) 3 (c) 2 (d) 1

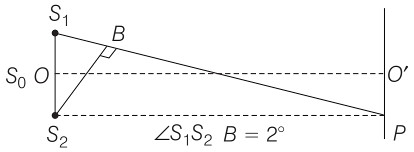
4 A mixture of light consisting of wavelength 590 nm and an unknown wavelength illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both light coincide. Further, it is observed that the third bright fringe is known light coincides with the 4th bright fringe of an unknown light. From this idea, the wavelength of an unknown light is

(a) 885.0 nm (b) 442.5 nm (c) 776.8 nm (d) 393.4 nm

5. In a Young's experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, then wavelength of monochromatic light used would be

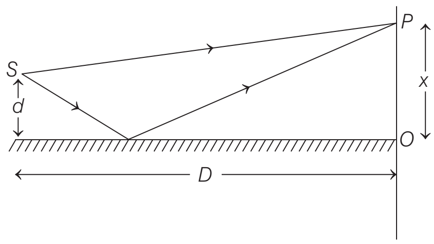
(a) 60×10^4 cm (b) 10×10^{-4} cm
(c) 10×10^{-5} cm (d) 6×10^{-5} cm

6 In the given figure, O' is the position of first bright range towards right from OP is the position of 5th bright fringe on the other side of O with respect to O'. If wavelength of used light is 6000 Å, then value of S_1B will be



(a) 2.4×10^{-4} m (b) 2.4×10^{-2} m
(c) 2.4×10^{-3} m (d) 2.4×10^{-6} m

7 A narrow slit S transmitting light of wavelength λ is placed a distance d above a large plane mirror as shown. The light coming directly from the slit and that after reflection interfere at P on the screen placed at a distance D from the slit. What will be x, for which first maxima occurs?



(a) $\frac{4d}{\lambda D}$ (b) $\frac{\lambda D}{4d}$ (c) $\frac{2d}{D}$ (d) $\frac{D}{2d}$

8 In the Young's double slit experiment, the intensity of light at a point on the screen, where the path difference in λ is K (λ being the wavelength of light used). The intensity at a point, where the path difference is $\frac{\lambda}{4}$, will be

➜ CBSE AIPMT 2014

(a) K (b) $\frac{K}{4}$ (c) $\frac{K}{2}$ (d) zero

9 The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å, respectively. If x is the distance of 4th maxima from the central one, then

(a) $x_{blue} = x_{green}$ (b) $x_{blue} > x_{green}$
(c) $x_{blue} < x_{green}$ (d) $x_{blue} / x_{green} = 5460/4360$

10 Two slits in Young's experiment have widths in the ratio 1 : 25. The ratio of intensity at the maxima and minima in the interference pattern $\frac{I_{max}}{I_{min}}$ is

➜ CBSE AIPMT 2015

(a) $\frac{9}{4}$ (b) $\frac{121}{49}$
(c) $\frac{49}{121}$ (d) $\frac{4}{9}$

11 In Young's double slit experiment, when wavelength used is 6000 Å and the screen is 40 cm from the slits, the fringes are 0.012 cm wide. What is the distance between the slits?

(a) 0.024 cm (b) 2.4 cm
(c) 0.24 cm (d) 0.2 cm

12 In Young's double slit experiment using sodium light ($\lambda = 5898$ Å), 92 fringes are seen. If given colour ($\lambda = 5461$ Å) is used, how many fringes will be seen?

(a) 62 (b) 67
(c) 85 (d) 99

13 In Young's experiment, two coherent sources are 0.90 mm apart and fringes are observed at a distance of 1 m, if 2nd dark fringe is at 1 mm distance from central fringe, then wavelength of the monochromatic light will be

(a) 60×10^{-4} cm (b) 10×10^{-4} cm
(c) 10×10^{-5} cm (d) 6×10^{-5} cm

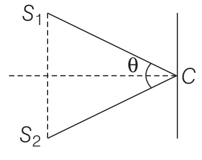
14 In Young's double slit experiment, the spacing between the slits is d and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distant screen is 1°, then value of d is

(a) 1 mm (b) 0.05 mm
(c) 0.03 mm (d) 0.01 mm

15 In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelength $\lambda_1 = 12000$ Å and $\lambda_2 = 10000$ Å. At what minimum distance from the common central bright fringe on the screen 2 cm from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other? ➜ NEET 2013

(a) 8 mm (b) 6 mm
(c) 4 mm (d) 3 mm

16 Young's double slit experimental arrangement is as shown in figure. If λ is the wavelength of light used and $\angle S_1CS_2 = \theta$, then the fringe width will be



(a) $\frac{\lambda}{\theta}$ (b) $\frac{\lambda}{2\theta}$ (c) $\lambda\theta$ (d) $\frac{2\lambda}{\theta}$

17 In Young's double slit experiment, the separation d between the slits is 2 mm, the wavelength λ of the light used is 5896 Å and distance D between the screen and slits is 100 cm. It is found that the angular width of the fringes is 0.20°. To increase the fringe angular width to 0.21° (with same λ and D) the separation between the slits needs to be changed to → NEET 2018

(a) 2.1 mm (b) 1.9 mm (c) 1.8 mm (d) 1.7 mm

18 In Young's double slit experiment, one of the slit is covered with a transparent sheet of thickness 3.6×10^{-3} cm due to which position of central fringe shifts to a position originally occupied by 30th bright fringe. The refractive index of the sheet, if $\lambda = 6000$ Å, is

(a) 1.5 (b) 1.2 (c) 1.3 (d) 1.7

19 In an experiment of double slits, interference fringes are obtained by using light of wavelength 4800 Å. If the first slit is covered with a thin sheet of glass having refractive index 1.4 and the second slit is covered with a sheet of same thickness of refractive index 1.7, then the central fringe is displaced to the position of 5th bright fringe. The thickness of glass will be

(a) 10.5×10^{-3} mm (b) 8×10^{-3} mm
(c) 6.5×10^{-3} mm (d) 2.5×10^{-3} mm

20 Young's double slit experiment is first performed in air and then in a medium other than air. It is found that 8th bright fringe in the medium lies, where 5th dark fringe lies in air. The refractive index of the medium is nearly → NEET 2017

(a) 1.25 (b) 1.59 (c) 1.69 (d) 1.78

21 What is necessary for easy occurrence of Fresnel's diffraction?

(a) Obstacle should of the order of wavelength
(b) Narrow opening should be of the order of wavelength
(c) Source and screen should be at finite distance from the obstacle
(d) All of the above

22 In Fraunhoffer diffraction, the centre of diffraction image is

(a) always bright (b) always dark
(c) sometimes bright and sometimes dark
(d) bright for large wavelength and dark for low wavelength

23 For Fraunhofer single slit diffraction?

(a) width of central maxima is proportional to λ
(b) on increasing the slit width, the width of central maxima decreases
(c) on making the slit width $a = \lambda$, central maxima spreads in the range $\pm$ 90°
(d) All of the above

24 For a parallel beam of monochromatic light of wavelength λ, diffraction is produced by a single slit whose width a is of the order of the wavelength of the light. If D is the distance of the screen from the slit, the width of the central maxima will be → CBSE AIPMT 2015

(a) $\frac{2D\lambda}{a}$ (b) $\frac{D\lambda}{a}$
(c) $\frac{DA}{\lambda}$ (d) $\frac{2Da}{\lambda}$

25 A beam of light $\lambda = 600$ nm from a distant source, falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on other side of the central bright fringe, is → CBSE AIPMT 2014

(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

26 At the first minimum adjacent to the central maximum of a single slit diffraction pattern, the phase difference between the Huygens' wavelet from the edge of the slit and the wavelet from the mid-point of the slit is → CBSE AIPMT 2015

(a) $\frac{\pi}{4}$ rad (b) $\frac{\pi}{2}$ rad
(c) π rad (d) $\frac{\pi}{8}$ rad

27 A linear aperture whose width is 0.02 cm is placed immediately in front of a lens of focal length 60 cm. The aperture is illuminated normally by a parallel beam of wavelength 5×10^{-5} cm. The distance of the first dark band of the diffraction pattern from the centre of the screen is → NEET 2016

(a) 0.10 cm (b) 0.25 cm
(c) 0.20 cm (d) 0.15 cm

28 Unpolarised light is incident from air on a plane surface of a material of refractive index μ. At a particular angle of incidence i, it is found that the reflected and refracted rays are perpendicular to each other. Which of the following options is correct for this situation? → NEET 2018

(a) $i = \sin^{-1}\left(\frac{1}{\mu}\right)$
(b) Reflected light is polarised with its electric vector perpendicular to the plane of incidence
(c) Reflected light is polarised with its electric vector parallel to the plane of incidence
(d) $i = \tan^{-1}\left(\frac{1}{\mu}\right)$

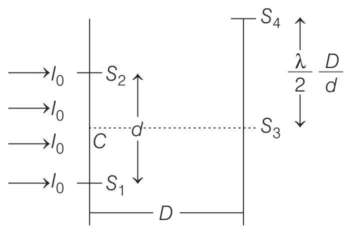
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 In Young's double slit experiment, the intensity at a point, where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to

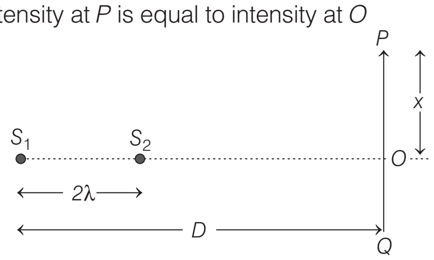
(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

2 In the given figure, C is middle point of line $S_1 S_2$. A monochromatic light of wavelength λ is incident on slits. The ratio of intensity of S_3 and S_4 is



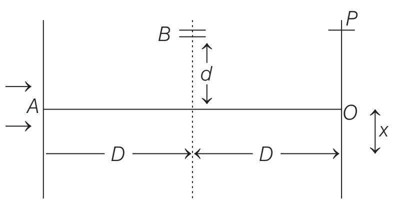
(a) 0 (b) ∞ (c) 4 : 1 (d) 1 : 4

3 Two coherent point sources S_1 and S_2, vibrating in phase emit light of wavelength λ. The separation between them is 2λ. The light is collected on a screen placed at a distance $D >> \lambda$ from the slit S_1 as shown. The minimum distance, so that intensity at P is equal to intensity at O



(a) $\sqrt{2}\,D$ (b) $\sqrt{3}\,D$ (c) $\sqrt{8}\,D$ (d) $\sqrt{5}\,D$

4 Consider the arrangement as shown. The distance D is large compared to d. Minimum value of d, so that there is a dark fringe at O, is



(a) $\sqrt{\frac{\lambda D}{2}}$ (b) $\sqrt{\frac{2}{D\lambda}}$ (c) $\sqrt{\frac{D}{2\lambda}}$ (d) $\sqrt{\frac{2\lambda}{D}}$

5 In a double slit experiment, the two slits are 1mm apart and the screen is placed 1 m away. A monochromatic light of wavelength 50 nm is used. What will be the width of each slit for obtaining ten maxima of double slit, within the central maxima of single slit pattern? ➔ CBSE AIPMT 2015

(a) 0.2 mm (b) 0.1 mm (c) 0.5 mm (d) 0.02 mm

6 Two beams of light of intensity I_1 and I_2 interfere to give an interference pattern. If the ratio of maximum intensity to that of minimum intensity is $\frac{25}{9}$, then $\frac{I_1}{I_2}$ is

(a) $\frac{5}{3}$ (b) 4

(c) $\frac{81}{625}$ (d) 16

7 Two polaroids P_1 and P_2 are placed with their axis perpendicular to each other. Unpolarised light I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its axis makes an angle 45° with that of P_1. The intensity of transmitted light through P_2 is ➔ NEET 2017

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$ (c) $\frac{I_0}{8}$ (d) $\frac{I_0}{16}$

8 The interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be ➔ NEET 2016

(a) $\frac{\sqrt{n}}{n+1}$ (b) $\frac{2\sqrt{n}}{n+1}$ (c) $\frac{\sqrt{n}}{(n+1)^2}$ (d) $\frac{2\sqrt{n}}{(n+1)^2}$

9 The intensity at the maximum in a Young's double slit experiment is I_0. Distance between two slits is $d = 5\lambda$, where λ is the wavelength of light used in the experiment. What will be the intensity in front of one of the slits on the screen placed at a distance $D = 10\ d$? ➔ NEET 2016

(a) $\frac{I_0}{4}$ (b) $\frac{3}{4}I_0$

(c) $\frac{I_0}{2}$ (d) I_0

10 In a diffraction pattern due to a single slit of width a, the first minimum is observed at an angle 30° when light of wavelength 5000 Å is incident on the slit. The first secondary maximum is observed at an angle of ➔ NEET 2016

(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$

ANSWERS

SESSION 1	**1** (c)	**2** (b)	**3** (c)	**4** (b)	**5** (d)	**6** (d)	**7** (b)	**8** (c)	**9** (c)	**10** (a)
	11 (d)	**12** (d)	**13** (d)	**14** (c)	**15** (b)	**16** (a)	**17** (b)	**18** (a)	**19** (b)	**20** (d)
	21 (c)	**22** (a)	**23** (d)	**24** (a)	**25** (d)	**26** (c)	**27** (d)	**28** (b)		
SESSION 2	**1** (a)	**2** (b)	**3** (b)	**4** (a)	**5** (a)	**6** (d)	**7** (c)	**8** (b)	**9** (c)	**10** (c)

Hints and Explanations

SESSION 1

1 $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I} + \sqrt{4I})^2 = 9I$

and $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$= (\sqrt{I} - \sqrt{4I})^2 = I$

2 Here, $a_1 = 8$, $a_2 = 6$

$$\because \frac{I_{\max}}{I_{\min}} = \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{8+6}{8-6}\right]^2 = \frac{49}{1}$$

3 Intensity at the centre of bright fringe,
$I_0 = I + I + 2\sqrt{II}\cos 0°$

$$I_0 = 2I + 2I \Rightarrow I_0 = 4I$$

Intensity at a point distant $\frac{P}{4}$

(with a phase difference $= \frac{2\pi}{4} = \frac{\pi}{2}$), is

$$I' = I + I + 2\sqrt{II}\cos\frac{\pi}{2}$$

$$\Rightarrow \quad I' = 2I + 2\sqrt{II} \times 0 = 2I$$

$$\therefore \quad \frac{I_0}{I'} = \frac{4I}{2I} = 2$$

4 As 4th bright fringe of an unknown wavelength λ' coincides with 3rd bright fringe of a known wavelength $\lambda = 590$ nm, therefore

$$\frac{4\lambda' D}{d} = \frac{3\lambda D}{d} \Rightarrow \lambda' = \frac{3}{4}\lambda = \frac{3}{4} \times 590$$

$$= 442.5 \text{ nm}$$

5 Distance of nth dark fringe from central fringe,

$$x_n = \frac{(2n-1)\lambda D}{2d}$$

$$\therefore \quad x_2 = \frac{(2 \times 2 - 1)\lambda D}{2d} = \frac{3\lambda D}{2d}$$

$$\Rightarrow 1 \times 10^{-3} = \frac{3 \times \lambda \times 1}{2 \times 0.9 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = 6 \times 10^{-5} \text{ cm}$$

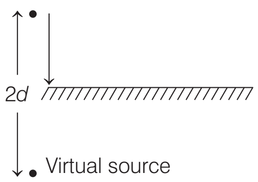
6 O' is the 4th fringe with respect to O.
i.e. $n = 4$, $d = 6000 \times 10^{-10}$ m …(i)

Path difference, $\Delta = S_1B = n\lambda$ …(ii)

From Eqs. (i) and (ii), we get

$\Delta = 4 \times 6 \times 10^{-7} = 2.4 \times 10^{-6}$ m

7 $x_n = \frac{(2n-1)\lambda D}{2d}$



Put $n = 1$ and $d = 2d$ as image of S will be $2d$ apart, we get

$$\therefore \quad x_1 = \frac{\lambda D}{2(2d)} = \frac{\lambda D}{4d}$$

8 Path difference λ means maxima,
$I_{\max} = K$

$$I = K\cos^2\frac{\phi}{2} = K\cos^2\left[\frac{2\pi}{\lambda} \times \frac{\lambda}{4} \times \frac{1}{2}\right]$$

$$= K\cos^2\frac{\pi}{4} = \frac{K}{2}$$

9 Distance of nth maxima, $x = n\lambda\frac{D}{d} \propto \lambda$

As, $\lambda_b < \lambda_g$; $\therefore$ $x_{\text{blue}} < x_{\text{green}}$

10 Given, YDSE experiment, having two slits of width are in the ratio of 1 : 25.

So, ratio of intensity,

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{1}{25} \quad \Rightarrow \quad \frac{I_2}{I_1} = \frac{25}{1}$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2}{(\sqrt{I_2} - \sqrt{I_1})^2} = \left[\frac{\sqrt{\frac{I_2}{I_1}} + 1}{\sqrt{\frac{I_2}{I_1}} - 1}\right]^2$$

$$\Rightarrow \left[\frac{5+1}{5-1}\right]^2 = \left(\frac{6}{4}\right)^2 = \frac{36}{16} = \frac{9}{4}$$

Thus, $\frac{I_{\max}}{I_{\min}} = \frac{9}{4}$

11 $\therefore$ $\beta = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\beta}$

$$= \frac{6000 \times 10^{-10} \times (40 \times 10^{-2})}{0.012 \times 10^{-2}} = 0.2 \text{ cm}$$

12 $\therefore$ $n_1\lambda_1 = n_2\lambda_2$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$\Rightarrow \quad \frac{n_1}{92} = \frac{5898}{5461} \Rightarrow n_1 = 99$$

13 Given, $d = 0.90 \times 10^{-3}$ m, $D = 1$ m

$$\left(\frac{3}{2}\right)\bar{X} = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\Rightarrow \quad \bar{X} = \frac{D\lambda}{d}, = \frac{2 \times 10^{-3}}{3}$$

$$\text{or} \quad \lambda = \frac{2 \times 10^{-3} \times 0.9 \times 10^{-3}}{3 \times 1}$$

$$= 6 \times 10^{-7} \text{ m} = 6 \times 10^{-5} \text{ cm}$$

14 $\sin\theta \approx \theta = \frac{Y}{D} \Rightarrow \Delta\theta = \frac{\Delta Y}{D}$

Angular fringe width, $\theta_0 = \Delta\theta$ (width $\Delta Y = \beta$)

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d} = 1° = \frac{\pi}{180} \text{ rad}$$

$\lambda = 6 \times 10^{-7}$ m

$$d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7}$$

$$= 3.44 \times 10^{-5} \text{ m} = 0.03 \text{ mm}$$

15 Given, $\lambda_1 = 12000$ Å, $\lambda_2 = 10000$ Å,
$D = 2$ cm and $d = 2$ mm $= 2 \times 10^{-3}$ cm

We have, $\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2} = \frac{12000}{10000} = \frac{6}{5}$

As, $x = \frac{n_1\lambda_1 D}{d}$

$$= \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$$

$$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3$$

$= 6$ mm

16 Fringe width, $\beta = \frac{D\lambda}{d}$

But here, $\theta = \frac{d}{D} \Rightarrow d = D\theta$

$$\therefore \quad \beta = \frac{D\lambda}{D\theta} = \frac{\lambda}{\theta}$$

17 In a YDSE, angular width of a fringe is given as

$$\theta = \frac{\lambda}{d} \Rightarrow \theta \propto \frac{1}{d} \quad \text{or} \quad \frac{\theta_1}{\theta_2} = \frac{d_2}{d_1} \qquad \ldots\text{(i)}$$

Here, $\theta_1 = 0.20°$, $\theta_2 = 0.21°$, $d_1 = 2$ mm

Substituting the given values in Eq. (i), we get

$$\frac{0.20°}{0.21°} = \frac{d_2}{2\text{ mm}}$$

$$\Rightarrow \quad d_2 = 2 \times \frac{0.20}{0.21} = \frac{0.40}{0.21} = 1.9\text{ mm}$$

18 The position of 30th bright fringe,

$$Y_{30} = \frac{30\lambda D}{d}$$

Position shift of central fringe is

$$Y_0 = \frac{30\lambda D}{d} \Rightarrow Y_0 = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow \frac{30\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow (\mu - 1) = \frac{30\lambda}{t}$$

$$(\mu - 1) = \frac{30 \times (6000 \times 10^{-10})}{3.6 \times 10^{-5}} = 0.5$$

$$\mu = 1.5$$

19 Displacement of central fringe

$$= (\mu_1 - 1)t\left(\frac{\beta}{\lambda}\right) - (\mu_2 - 1)t\left(\frac{\beta}{\lambda}\right) = \frac{5}{\beta}$$

$$\therefore \quad (\mu_1 - \mu_2)t\left(\frac{\beta}{\lambda}\right) = 5\beta$$

$$(0.3)t\left(\frac{1}{4800 \times 10^{-10}}\right) = 5$$

$$t = \frac{4.8 \times 10^{-7} \times 5}{0.3}$$

$$\Rightarrow t = 8 \times 10^{-6}\text{ m} = 8 \times 10^{-3}\text{ mm}$$

20 According to question, 5th dark fringe in air = 8 bright fringe in the medium

$$(2 \times 5 - 1)\frac{\lambda D}{2d} = 8\frac{\lambda D}{\mu d}$$

$$\Rightarrow 9\frac{\lambda D}{2d} = 8\frac{\lambda D}{\mu d} \Rightarrow \frac{9}{2} = \frac{8}{\mu} \Rightarrow \mu = \frac{8 \times 2}{9}$$

$\therefore$ Refractive index of the medium,

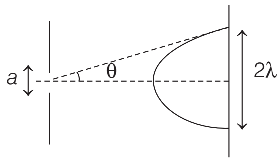
$$\mu = \frac{16}{9} = 1.7777 \approx 1.78$$

21 If either source or screen, both are at finite distance from the diffracting device, the diffraction is called Fresnel diffraction.

22 In Fraunhofer diffraction, the centre of diffraction image is always bright.

23 For Fraunhofer diffraction of a single slit, we have all the conditions true, i.e. central maxima has width $\propto \lambda$, on increasing the slit width, the width of central maxima decreases. Also, if $a \approx \lambda$, then central maxima has angular separation of $\pm\ 90°$.

24 $\sin\theta = \dfrac{\lambda}{a}$



$$\frac{y}{D} = \frac{\lambda}{a} \Rightarrow y = \frac{\lambda D}{a}$$

Width of central maxima is $\dfrac{2\lambda D}{a}$.

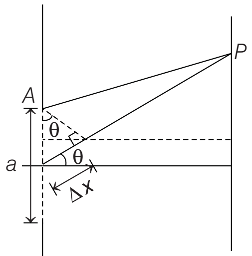
25 Distance between first order dark fringes = Width of principal maxima

$$x = \frac{2\lambda D}{d} = \frac{2 \times 600 \times 10^{-9} \times 2}{10^{-3}}$$

$$= 2400 \times 10^{-6}$$

$$= 2.4 \times 10^{-3}\text{ m} = 2.4\text{ mm}$$

26 For first minima at P, $a\sin\theta = n\lambda$



where, $\quad n = 1 \Rightarrow a\sin\theta = \lambda$

So, phase difference,

$$\Delta\phi_1 = \frac{\Delta x_1}{\lambda} \times 2\pi = \frac{(a/2)\sin\theta}{\lambda} \times 2\pi$$

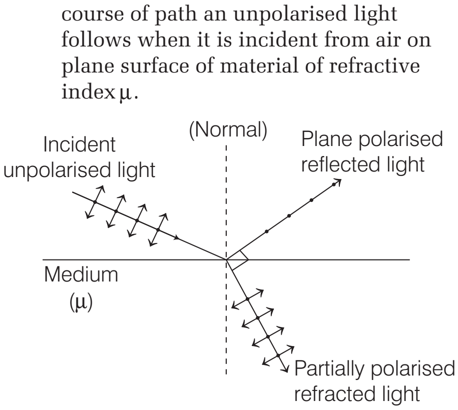
$$= \frac{\lambda}{2\lambda} \times 2\pi = \pi\text{ rad}$$

27 *Key Idea* First minima is formed at a distance

$$Y = \frac{\lambda D}{a} \Rightarrow Y = \frac{(5 \times 10^{-5})\,(0.6)}{0.02 \times 10^{-2}}\text{(given)}$$

$$\Rightarrow Y = 0.15\text{ cm}$$

28 The figure shown below represents the course of path an unpolarised light follows when it is incident from air on plane surface of material of refractive index μ.



When the beam of unpolarised light is reflected from a medium (refractive index = μ) and if reflected and refracted light are perpendicular to each other. Then, the reflected light is completely plane polarised at a certain angle of incidence. This means, the reflected light has electric vector perpendicular to incidence plane.

SESSION 2

1 $\therefore \quad \phi = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{6} = \dfrac{\pi}{3}$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I' = I + I + 2I\cos\frac{\pi}{3} = 3I$$

and $I_0 = I + I + 2I\cos 0° = 4I$

$$\therefore \quad \frac{I'}{I_0} = \frac{3}{4}$$

2 At S_3, $\Delta x = S_1S_3 - S_2S_3 = 0$

$$\therefore \quad \phi = \frac{2\pi}{\lambda}\Delta x = 0$$

$$I_3 = I_0 + I_0 + 2\sqrt{I_0 \times I_0}\cos 0°$$

$$I_3 = 4I_0$$

The path difference at S_4 is

$$\Delta x' = S_1S_4 - S_2S_4 = \frac{dY}{D}$$

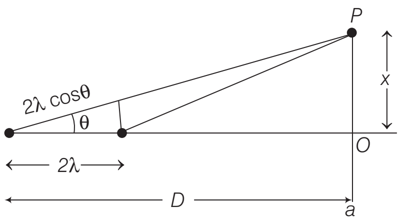
$$= \frac{d}{D} \times \frac{\lambda D}{2d} = \frac{\lambda}{2} \qquad \left[\because\ Y = \frac{\lambda D}{2d}\right]$$

$$\phi' = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

$$I_4 = I_0 + I_0 + 2I_0\cos\pi = 0$$

$$\frac{I_3}{I_4} = \frac{4I_0}{0} = \infty$$

3 Path difference $= 2\lambda\cos\theta$



$$2\lambda\cos\theta = n\lambda$$

For x to be minimum, $n = 1$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60°,\ x = D\tan 60° = \sqrt{3}\,D$$

4 Path difference $= AB + BO - 2D$

$$\Rightarrow \quad 2\sqrt{(D^2 + d^2)} - 2D = \frac{\lambda}{2}$$

$$\Rightarrow \quad 2\sqrt{D^2 + d^2} = \frac{\lambda}{2} + 2D$$

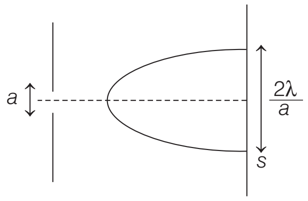
$$4(D^2 + d^2) = \frac{\lambda^2}{4} + 4D^2 + 2\lambda D$$

Eliminating $\dfrac{\lambda^2}{4}$ as

$$\lambda << D,\ d = \sqrt{\frac{\lambda D}{2}}$$

5 Given, $d = 1 \text{ mm} = 1 \times 10^{-3}$ m,

$D = 1$ m, $\lambda = 50 \times 10^{-9}$ m



Width of central maxima

= Width of ten maxima

$$\frac{2\lambda}{a} = 10\left(\frac{\lambda D}{d}\right)$$

$$\Rightarrow \quad a = \frac{2d}{10D} = \frac{2 \times 10^{-3}}{10 \times 1}$$

$$\Rightarrow \quad a = 2 \times 10^{-4} \text{ m}$$

$$a = 0.2 \text{ mm}$$

6 $\because \dfrac{I_{\max}}{I_{\min}} = \dfrac{25}{9}$ or $\left(\dfrac{a_1 + a_2}{a_1 - a_2}\right)^2 = \dfrac{25}{9}$

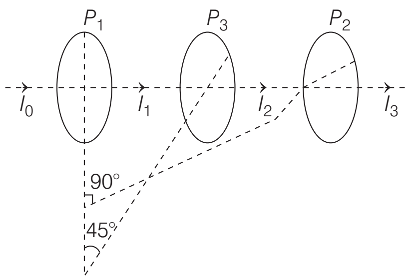
where, a denotes amplitude.

or $\dfrac{a_1 + a_2}{a_1 - a_2} = \dfrac{5}{3}$ or $\dfrac{a_1}{a_2} = 4$

As, $(\text{amplitude})^2 \propto$ intensity

Hence, $\dfrac{I_1}{I_2} = \left(\dfrac{a_1}{a_2}\right)^2 = 16$

7 According to the question,



From the above diagram, intensity transmitted through P_3,

$$I_2 = \frac{I_0}{2}\cos^2 45°$$

$$\Rightarrow \quad I_2 = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow I_2 = \frac{I_0}{4}$$

Similarly, intensity transmitted through P_2,

$$I_3 = \frac{I_0}{4}\cos^2 45° \quad \Rightarrow \quad I_3 = \frac{I_0}{4} \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad I_3 = \frac{I_0}{4} \times \frac{1}{2} \quad \Rightarrow I_3 = \frac{I_0}{8}$$

8 It is given that,

$$\frac{I_2}{I_1} = n \quad \Rightarrow \quad I_2 = nI_1$$

$\therefore$ Ratio of intensities is given by

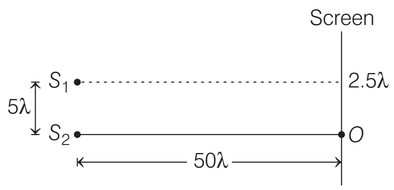
$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2 - (\sqrt{I_2} - \sqrt{I_1})^2}{(\sqrt{I_1} + \sqrt{I_2})^2 + (\sqrt{I_2} - \sqrt{I_1})^2)}$$

$$= \frac{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 - \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 + \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}$$

$$= \frac{(\sqrt{n} + 1)^2 - (\sqrt{n} - 1)^2}{(\sqrt{n} + 1)^2 + (\sqrt{n} - 1)^2}$$

$$= \frac{2\sqrt{n}}{n + 1}$$

9



According to question, the intensity at maximum in this Young's double slit experiment is I_0.

$$\Rightarrow \quad I_{\max} = I_0$$

$\because$ Path difference

$$= \frac{dY_n}{D} = \frac{d \times \frac{d}{2}}{10d} = \frac{d}{20} = \frac{\lambda}{4} \qquad [\because d = 5\lambda]$$

A path difference of λ corresponds to phase difference 2π.

So, for path difference $\lambda/4$, phase difference,

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2 = 90°$$

As we know, $I = I_0 \cos^2 \dfrac{\phi}{2}$

$$\Rightarrow \quad I = I_0 \cos^2 \frac{90°}{2}$$

$$\Rightarrow \quad I = I_0 \times \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow I = \frac{I_0}{2}$$

10 As the first minimum is observed at an angle of 30° in a diffraction pattern due to a single slit of width a.

i.e. $n = 1$, $\theta = 30°$

$\because$ According to Bragg's law of diffraction,

$a \sin\theta = n\lambda \Rightarrow a \sin 30° = (1)\,\lambda$

$[\because n = 1]$

$$\Rightarrow \quad a = 2\lambda \quad \ldots\text{(i)} \quad \left[\because \sin 30° = \frac{1}{2}\right]$$

For first secondary maxima,

$$a \sin\theta_1 = \frac{3\lambda}{2} \Rightarrow \sin\theta_1 = \frac{3\lambda}{2a} \qquad \ldots\text{(ii)}$$

Substituting the value of a from Eq. (i) into Eq. (ii), we get

$$\sin\theta_1 = \frac{3\lambda}{4\lambda} \quad \Rightarrow \quad \sin\theta_1 = \frac{3}{4}$$

$$\Rightarrow \theta_1 = \sin^{-1}\left(\frac{3}{4}\right)$$